١,٥-

SECOND YEAR CIVIL

STRUCTURAL ANALYSIS

Part (1) د/ صالح د/ نصر

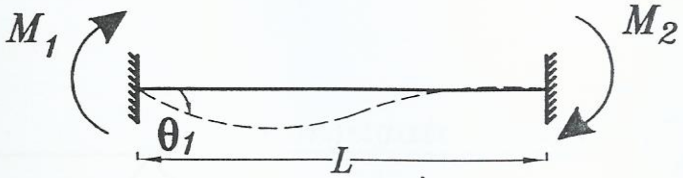
CONSISTENT DEFORMATIONS (2)

FIXED END MOMENT PROOFS

3

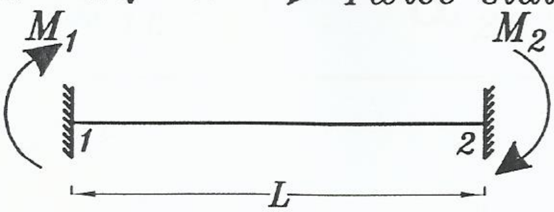
Example:

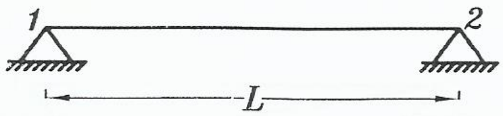
Find M_1 & M_2 in terms of θ_1.



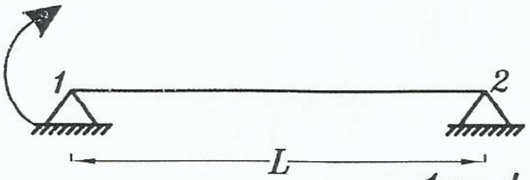
$UN = 4 \qquad EQ = 2 \longrightarrow UN > EQ \longrightarrow$ *Indeterminate*

$UN - EQ = 2 \longrightarrow$ *Twice statically indeterminate*

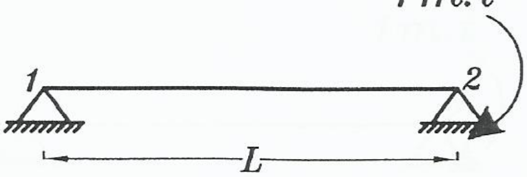




Main system(0)



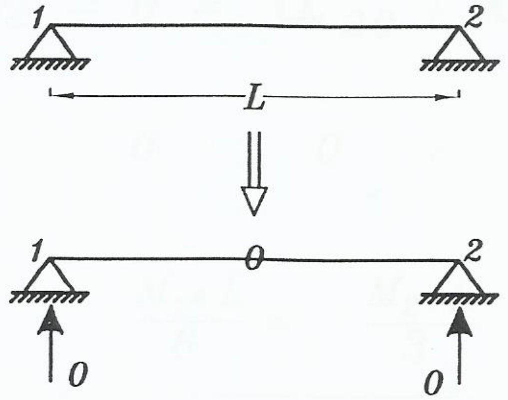
Correction system(1) $x\,M_1$



Correction system(2) $x\,M_2$

$$\theta_1 = \alpha_{10} + \alpha_{11}(M_1) + \alpha_{12}(M_2)$$

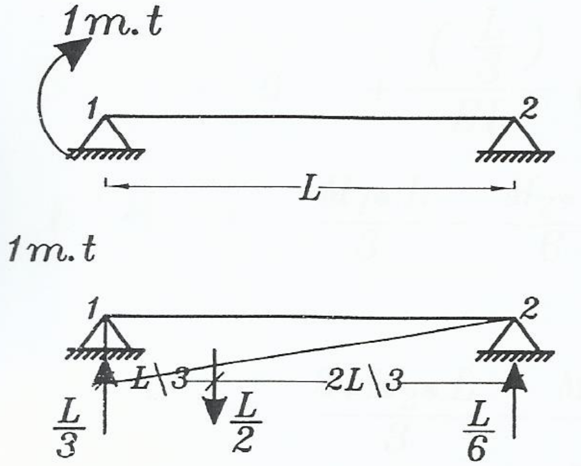
$$\theta_2 = 0 = \alpha_{20} + \alpha_{21}(M_1) + \alpha_{22}(M_2)$$



Main system(0)

Conjugate Beam $\left(/EI\right)$

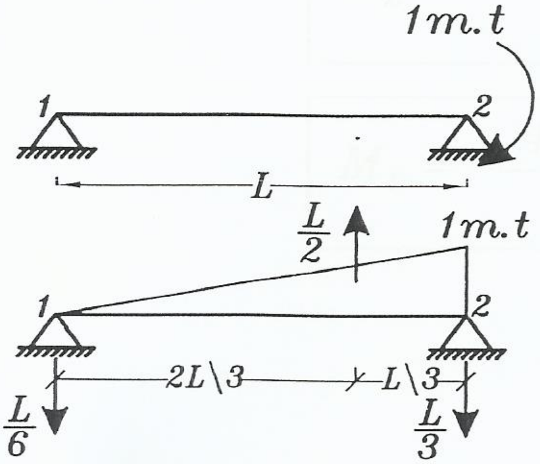
$$\alpha_{10} = 0 \qquad \alpha_{20} = 0$$



Correction system(1) $\left(x\, M_1\right)$

Conjugate Beam $\left(/EI\right)$

$$\alpha_{11} = \frac{1}{EI} * \frac{L}{3} \qquad \alpha_{21} = -\frac{1}{EI} * \frac{L}{6}$$



Correction system(2) $\left(x\, M_2\right)$

Conjugate Beam $\left(/EI\right)$

$$\alpha_{12} = \frac{1}{EI} * -\frac{L}{6} \qquad \alpha_{22} = \frac{1}{EI} * \frac{L}{3}$$

$$\theta_2 = 0 = \alpha_{20} + \alpha_{21}(M_1) + \alpha_{22}(M_2)$$

$$0 = 0 + \frac{(-\frac{L}{6})}{EI}(M_1) + \frac{(\frac{L}{3})}{EI}(M_2)$$

$$\frac{M_1 * L}{6} = \frac{M_2 * L}{3} \Longrightarrow \boxed{M_1 = 2\,M_2} \Longrightarrow EQ.1$$

$$\theta_1 = \alpha_{10} + \alpha_{11}(M_1) + \alpha_{12}(M_2)$$

$$\theta_1 = 0 + \frac{(\frac{L}{3})}{EI}(M_1) + \frac{(-\frac{L}{6})}{EI}(M_2)$$

$$EI\,\theta_1 = \frac{M_1 * L}{3} - \frac{M_2 * L}{6}$$

$$EI\,\theta_1 = \frac{2 * M_2 * L}{3} - \frac{M_2 * L}{6} = \frac{M_2 * L}{2} \Longrightarrow EQ.2$$

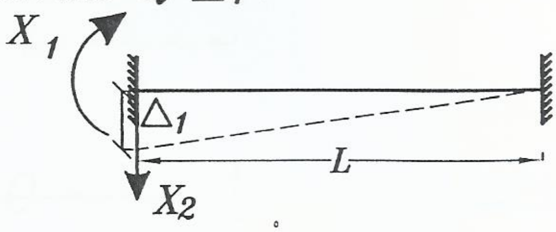
و بالتعويض من معادلة رقم ١ فى معادلة رقم ٢

$$\boxed{M_2 = \frac{2 * EI}{L}\,\theta_1}$$

$$\boxed{M_1 = \frac{4 * EI}{L}\,\theta_1}$$

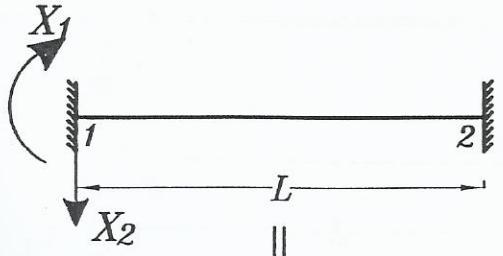
Example:

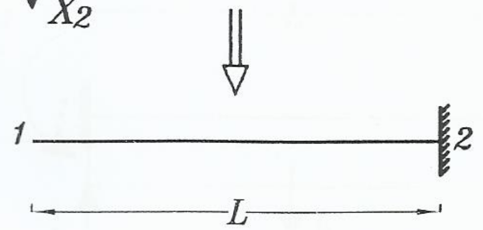
Find X_1 & X_2 in terms of $\triangle_1$.



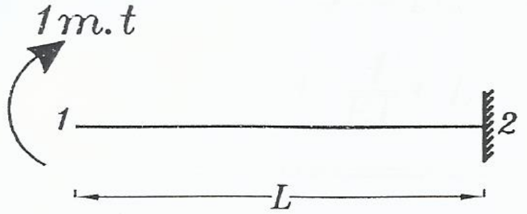
UN = 4 EQ = 2 $\longrightarrow$ UN > EQ $\longrightarrow$ Indeterminate

UN − EQ = 2 $\longrightarrow$ Twice statically indeterminate

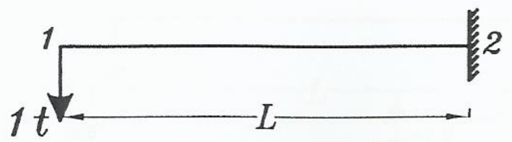




Main system(0)



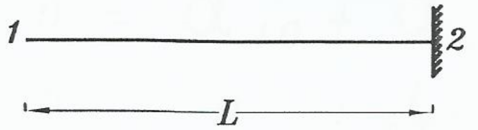
Correction system(1) $x\,X_1$



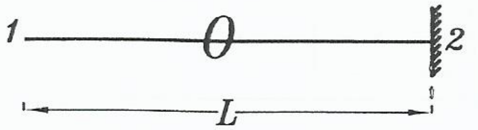
Correction system(2) $x\,X_2$

$$\triangle_1 = \delta_{10} + \delta_{11}(X_1) + \delta_{12}(X_2)$$

$$\theta_1 = 0 = \alpha_{10} + \alpha_{11}(X_1) + \alpha_{12}(X_2)$$

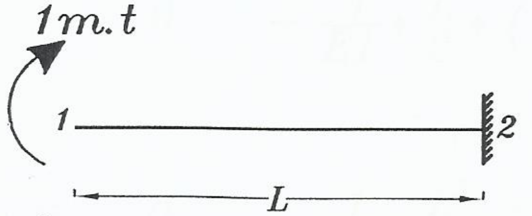


Main system(0)

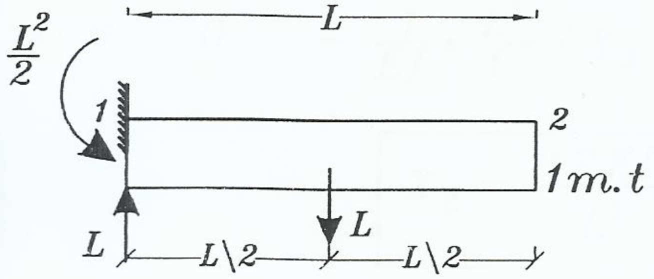


Conjugate Beam (/EI)

$$\alpha_{10} = 0 \qquad \delta_{10} = 0$$

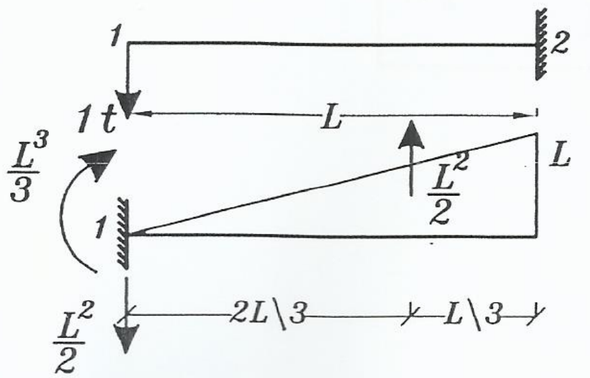


Correction system(1) ($x X_1$)



Conjugate Beam (/EI)

$$\alpha_{11} = + \frac{1}{EI} * L \qquad \delta_{11} = - \frac{1}{EI} * \frac{L^2}{2}$$



Correction system(2) ($x X_2$)

Conjugate Beam (/EI)

$$\alpha_{12} = - \frac{1}{EI} * \frac{L^2}{2} \qquad \delta_{12} = + \frac{1}{EI} * \frac{L^3}{3}$$

$$\theta_1 = 0 = \alpha_{10} + \alpha_{11}(X_1) + \alpha_{12}(X_2)$$

$$0 = 0 + \frac{1}{EI} * L * (X_1) - \frac{1}{EI} * \frac{L^2}{2} * (X_2)$$

$$\boxed{X_1 = \frac{L}{2} * (X_2)} \Longrightarrow EQ.1$$

$$\triangle_1 = \delta_{10} + \delta_{11}(X_1) + \delta_{12}(X_2)$$

$$\triangle_1 = 0 - \frac{1}{EI} * \frac{L^2}{2} * (X_1) + \frac{1}{EI} * \frac{L^3}{3} * (X_2) \Longrightarrow EQ.2$$

و بالتعويض من معادلة رقم ١ فى معادلة رقم ٢

$$\triangle_1 = 0 - \frac{1}{EI} * \frac{L^3}{4} * (X_2) + \frac{1}{EI} * \frac{L^3}{3} * (X_2)$$

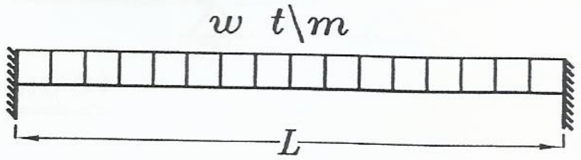
$$\boxed{X_2 = \frac{12EI}{L^3} \triangle_1}$$

$$X_1 = \frac{L}{2} * (X_2) = \frac{L}{2} * \frac{12EI}{L^3} \triangle_1$$

$$\boxed{X_2 = \frac{6\,EI}{L^2} \triangle_1}$$

Example:

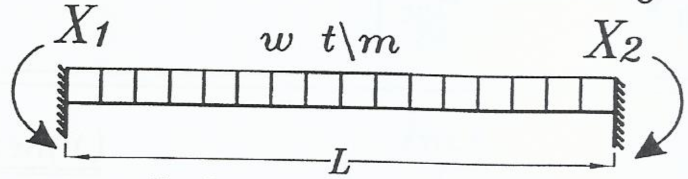
For the shown beam draw the B.M.D and calculate the reactions



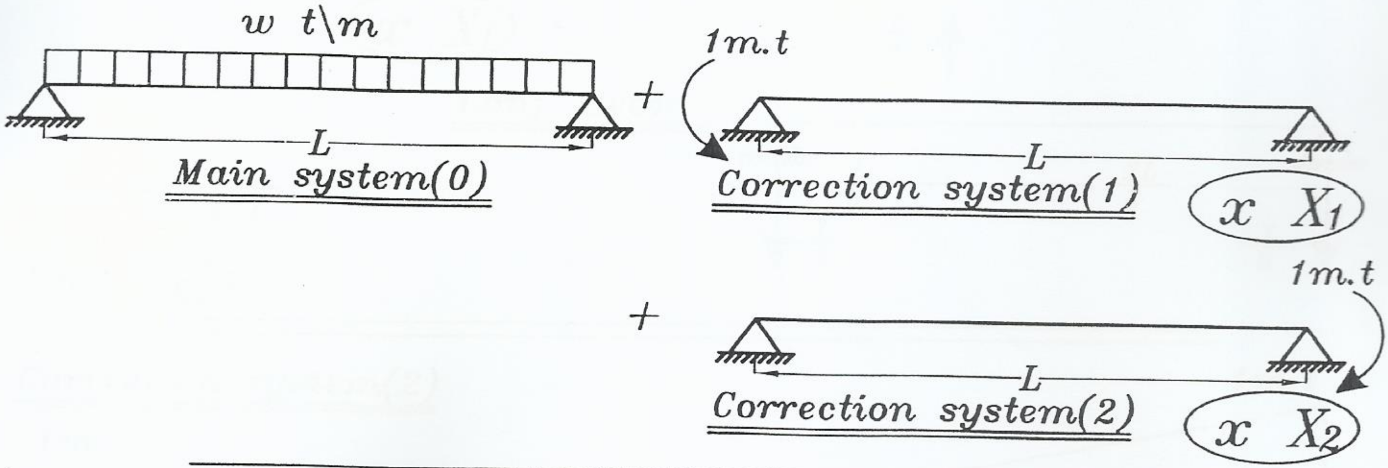
UN = 4 & EQ = 2 ; باهمال الX

EQ < UN ------ Indet. structure

UN − EQ = 4 − 2 = 2 ---- Twice statically indeterminate.



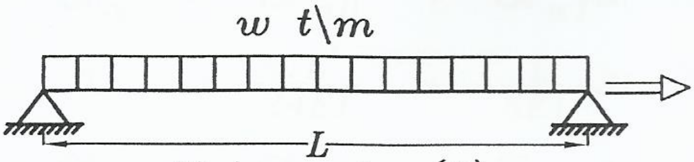
Indeterminate Beam



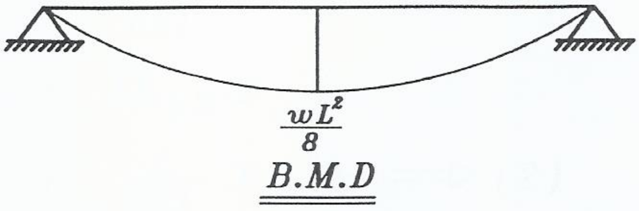
$$\alpha_a = \alpha_{a0} + \alpha_{a1} x X_1 + \alpha_{a2} x X_2$$

$$\alpha_b = \alpha_{b0} + \alpha_{b1} x X_1 + \alpha_{b2} x X_2$$

Main system(0)



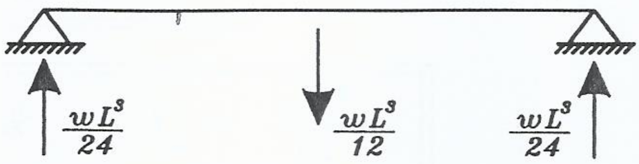
Main system(0)



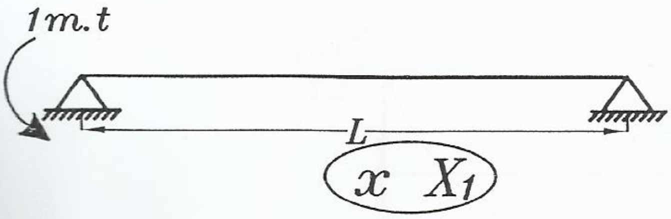
Conj. Beam

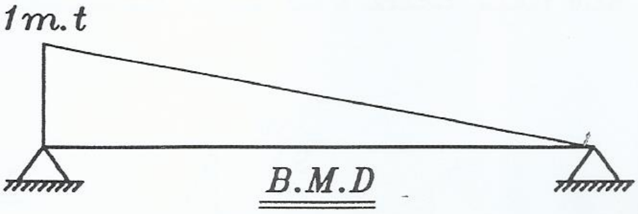
$$\alpha_{a0} = \frac{wL^3}{24EI}$$

$$\alpha_{b0} = -\frac{wL^3}{24EI}$$



Correction system(1)

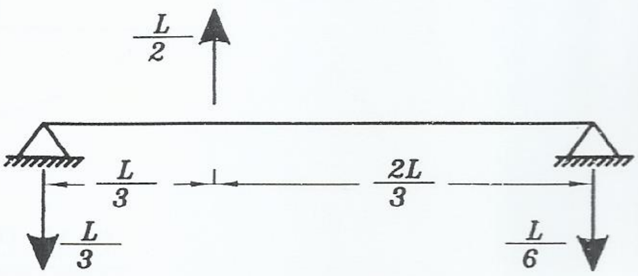




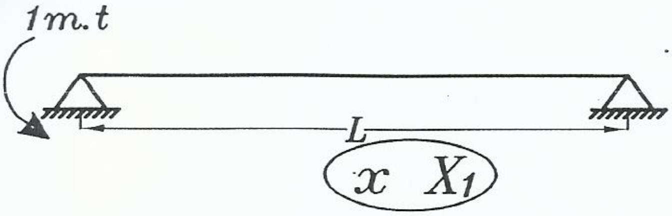
Conj. Beam

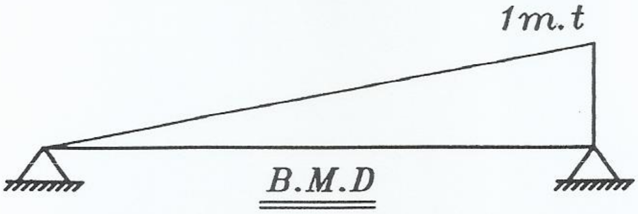
$$\alpha_{a1} = -\frac{L}{3EI}$$

$$\alpha_{b1} = \frac{L}{6EI}$$



Correction system(2)

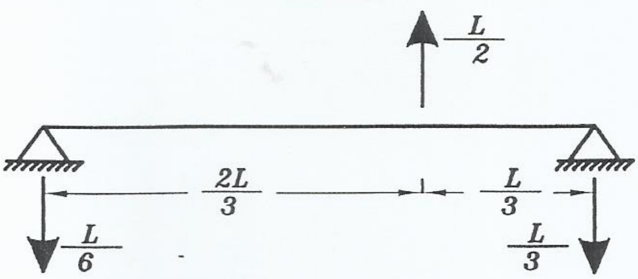




Conj. Beam

$$\alpha_{a2} = -\frac{L}{6EI}$$

$$\alpha_{b2} = \frac{L}{3EI}$$



$$\alpha_a = \alpha_{a0} + \alpha_{a1} x \; X_1 + \alpha_{a2} x \; X_2$$

$$\alpha_b = \alpha_{b0} + \alpha_{b1} x \; X_1 + \alpha_{b2} x \; X_2$$

$$\alpha_a = \alpha_{a0} + \alpha_{a1} x X_1 + \alpha_{a2} x X_2$$

$$0 = \frac{wL^3}{24EI} + -\frac{L}{3EI} x X_1 + -\frac{L}{6EI} x X_2 \Longrightarrow (1)$$

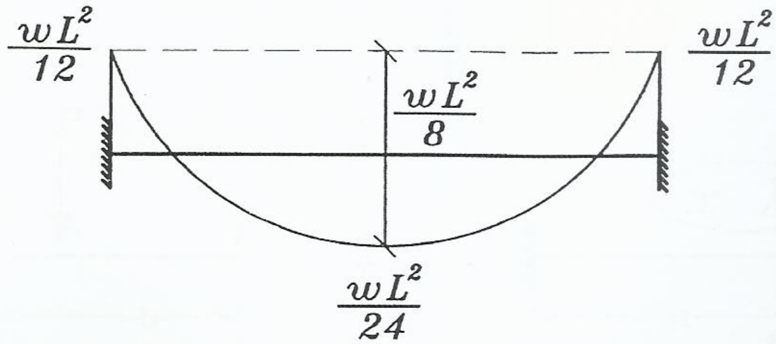
$$\alpha_b = \alpha_{b0} + \alpha_{b1} x X_1 + \alpha_{b2} x X_2$$

$$0 = -\frac{wL^3}{24EI} + \frac{L}{6EI} x X_1 + \frac{L}{3EI} x X_2 \Longrightarrow (2)$$

Solving the two equations :

$$\boxed{X_1 = \frac{wL^2}{12} \quad \& \quad X_2 = \frac{wL^2}{12}}$$

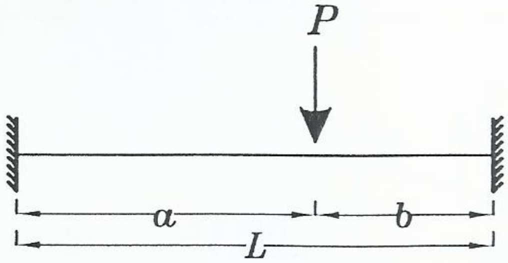
و هذه القيم تحفظ لاننا سوف نستخدمها فى درس أخر

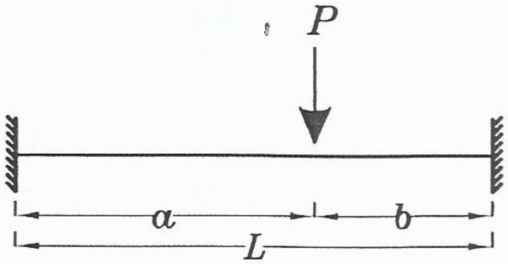


Final B.M.D

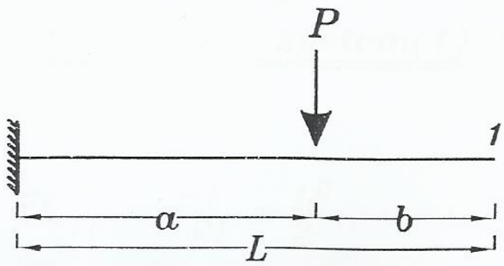
Example:

For the shown beam draw the B.M.D and calculate the reactions



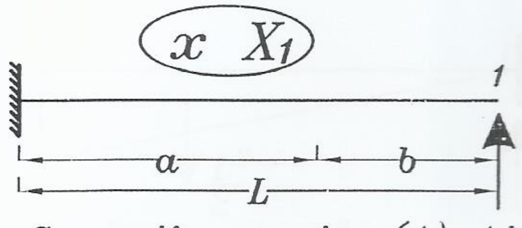


Indeterminate Beam



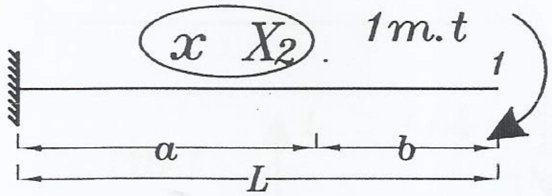
Main system(0)

+



Correction system(1) 1t

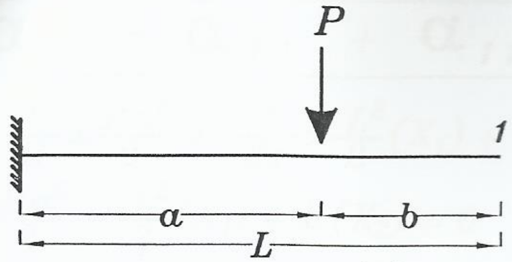
+



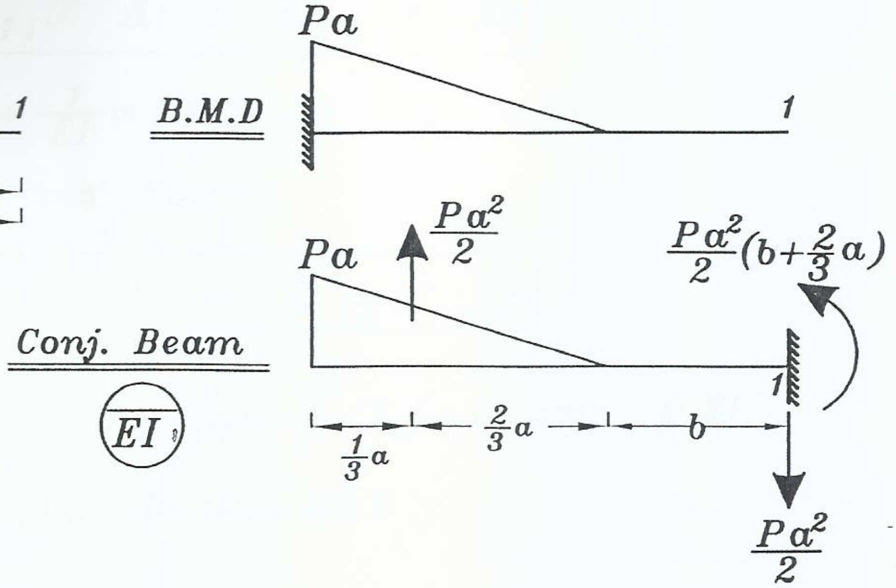
Correction system(2)

$$\alpha_1 = \alpha_{10} + \alpha_{11} x\ X_1 + \alpha_{12} x\ X_2$$

$$\delta_1 = \delta_{10} + \delta_{11} x\ X_1 + \delta_{12} x\ X_2$$

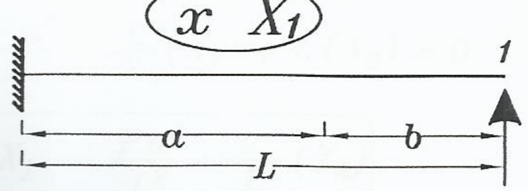


Main system(0)

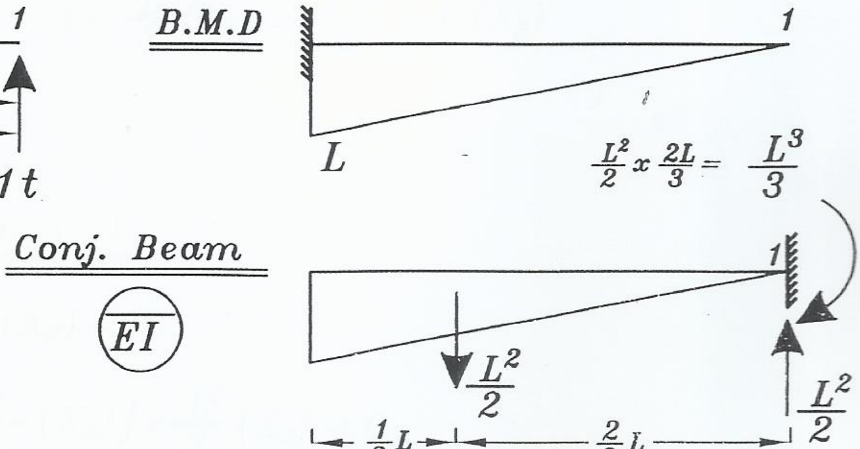


$$\alpha_{10} = \frac{1}{EI} x \frac{Pa^2}{2}$$

$$\delta_{10} = \frac{1}{EI} x \frac{Pa^2}{2}(b+\frac{2}{3}a)$$

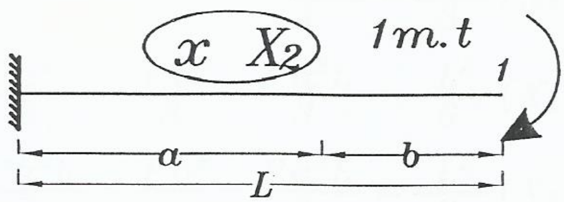


Correction system(1) 1t

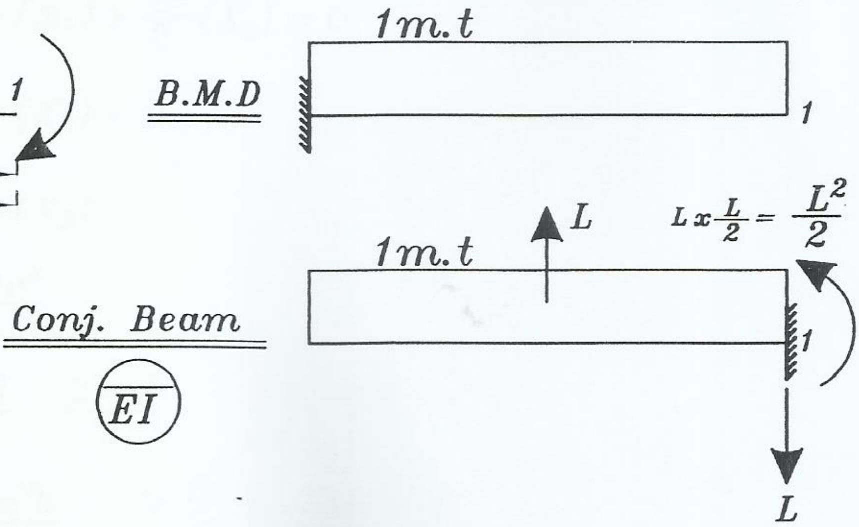


$$\alpha_{11} = \frac{-1}{EI} x \frac{L^2}{2}$$

$$\delta_{11} = \frac{-1}{EI} x \frac{L^3}{3}$$



Correction system(2)



$$\alpha_{12} = \frac{1}{EI} x \ L$$

$$\delta_{12} = \frac{1}{EI} x \frac{L^2}{2}$$

$$\boxed{\alpha_1 = \alpha_{10} + \alpha_{11} x\ X_1 + \alpha_{12} x\ X_2}$$

$$\frac{1}{EI} x \frac{P a^2}{2} + \frac{-1}{EI} x \frac{L^2}{2}(X_1) + \frac{1}{EI} x\ L\,(X_2) = 0 \Longrightarrow\quad x\ EI$$

$$\frac{P a^2}{2} - \frac{L^2}{2}(X_1) + L\,(X_2) = 0 \Longrightarrow EQ.1$$

$$\boxed{\delta_1 = \delta_{10} + \delta_{11} x\ X_1 + \delta_{12} x\ X_2}$$

$$\frac{1}{EI} x \frac{P a^2}{2}(b+\tfrac{2}{3}a) + \frac{-1}{EI} x \frac{L^3}{3}(X_1) + \frac{1}{EI} x \frac{L^2}{2}(X_2) = 0 \Longrightarrow\quad x\ EI$$

$$\frac{P a^2}{2}(b+\tfrac{2}{3}a) - \frac{L^3}{3}(X_1) + \frac{L^2}{2}(X_2) = 0 \Longrightarrow EQ.2$$

From equation (1)

$$\frac{P a^2}{2} - \frac{L^2}{2}(X_1) + L\,(X_2) = 0 \Longrightarrow \frac{L^2}{2}(X_1) = \frac{P a^2}{2} + L\,(X_2)$$

$$\boxed{X_1 = \frac{P a^2}{L^2} + \frac{2}{L}(X_2)}$$

Substitute in equation (2)

$$\frac{P a^2}{2}(b+\tfrac{2}{3}a) - \frac{L^3}{3}(X_1) + \frac{L^2}{2}(X_2) = 0$$

$$\frac{P a^2}{2}(b+\tfrac{2}{3}a) - \frac{L^3}{3}\left[\frac{P a^2}{L^2} + \frac{2}{L}(X_2)\right] + \frac{L^2}{2}(X_2) = 0$$

$$\frac{P a^2 b}{2} + \frac{P a^3}{3} - \frac{P a^2 L}{3} - \frac{2 L^2}{3}(X_2) + \frac{L^2}{2}(X_2) = 0$$

$$\frac{P a^2 b}{2} + \frac{P a^3}{3} - \frac{P a^2 L}{3} - \frac{L^2}{6}(X_2) = 0$$

$$\frac{P a^2 b}{2} + \frac{P a^3}{3} - \frac{P a^2 L}{3} = \frac{L^2}{6}(X_2)$$

$$X_2 = \frac{3 P a^2 b}{L^2} + \frac{2 P a^3}{L^2} - \frac{2 P a^2}{L}$$

$$= \frac{3 P a^2 b}{L^2} + \frac{2 P a^2 (L-b)}{L^2} - \frac{2 P a^2}{L}$$

$$= \frac{3 P a^2 b}{L^2} + \cancel{\frac{2 P a^2}{L}} - \frac{2 P a^2 b}{L^2} - \cancel{\frac{2 P a^2}{L}}$$

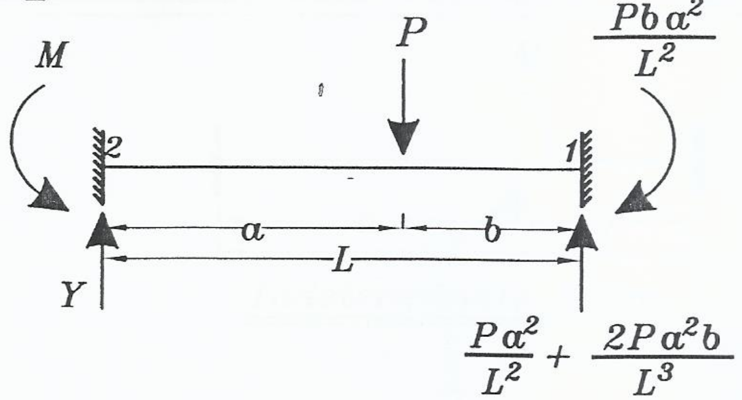
$$= \frac{P a^2 b}{L^2} \qquad X_2 = \frac{P a^2 b}{L^2}$$

Substitute in equation (1)

$$X_1 = \frac{P\alpha^2}{L^2} + \frac{2}{L}(X_2)$$

$$= \frac{P\alpha^2}{L^2} + \frac{2}{L}\left[\frac{P\alpha^2 b}{L^2}\right]$$

$$= \frac{P\alpha^2}{L^2} + \frac{2P\alpha^2 b}{L^3}$$



$\Sigma M@2 = 0$

$$\frac{Pb\alpha^2}{L^2} + P x \alpha - \left[\frac{P\alpha^2}{L^2} + \frac{2P\alpha^2 b}{L^3}\right] x L - M = 0$$

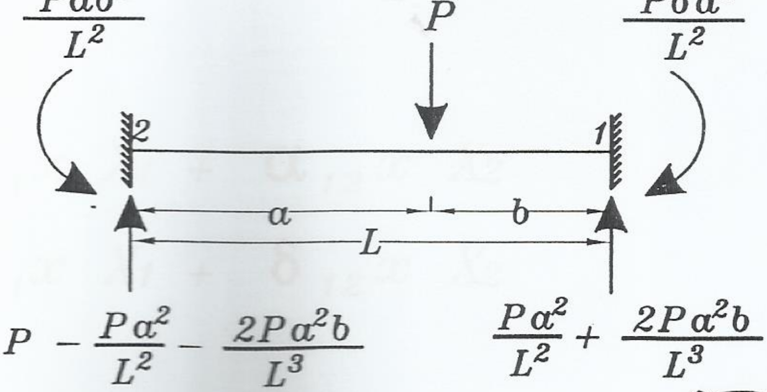
$$\frac{Pb\alpha^2}{L^2} + P x \alpha - \frac{P\alpha^2}{L} - \frac{2P\alpha^2 b}{L^2} - M = 0$$

$$-\frac{Pb\alpha^2}{L^2} + \frac{P\alpha L^2}{L^2} - \frac{P\alpha^2 L}{L^2} = M$$

$$-\frac{Pb\alpha^2}{L^2} + \frac{P\alpha(\alpha+b)^2}{L^2} - \frac{P\alpha^2(\alpha+b)}{L^2} = M$$

$$-\cancel{\frac{Pb\alpha^2}{L^2}} + \frac{\cancel{P\alpha^3} + P\alpha b^2 + \cancel{2Pb\alpha^2}}{L^2} - \cancel{\frac{P\alpha^3}{L^2}} - \cancel{\frac{Pb\alpha^2}{L^2}} = M$$

$$M = \frac{P\alpha b^2}{L^2}$$



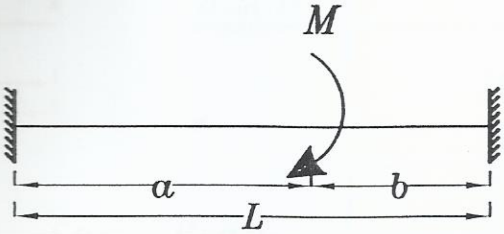
$\Sigma Y = 0$

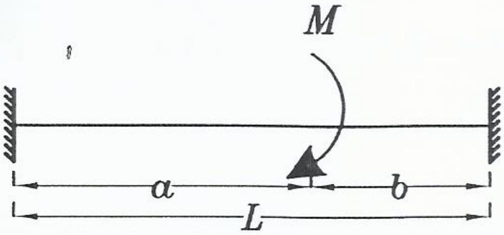
$$\frac{P\alpha^2}{L^2} + \frac{2P\alpha^2 b}{L^3} + Y - P = 0$$

$$Y = P - \frac{P\alpha^2}{L^2} - \frac{2P\alpha^2 b}{L^3}$$

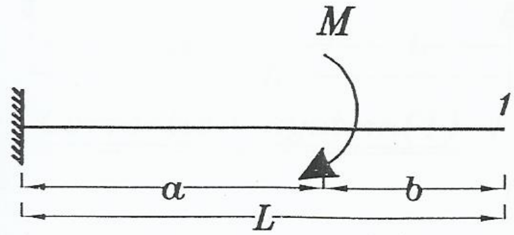
Example:

For the shown beam draw the B.M.D and calculate the reactions



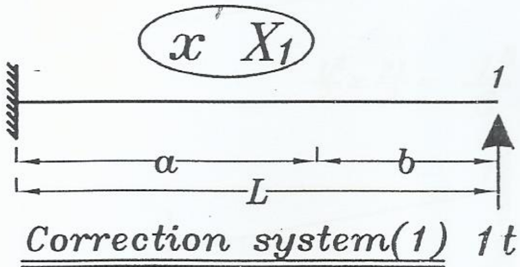


Indeterminate Beam



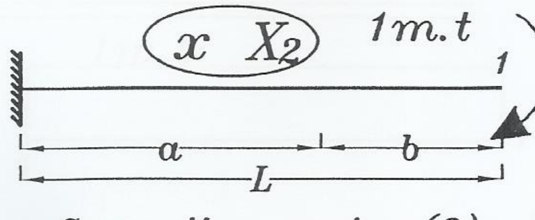
Main system(0)

+



Correction system(1)

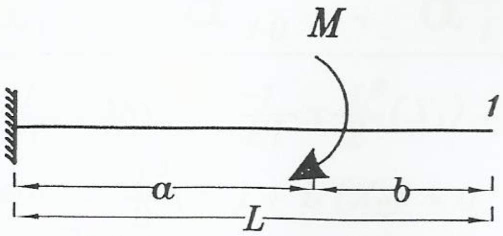
+



Correction system(2)

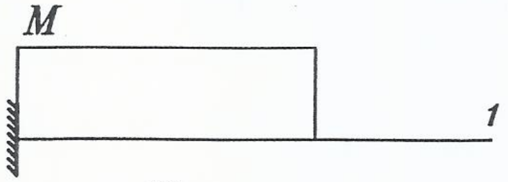
$$\alpha_1 = \alpha_{10} + \alpha_{11} x X_1 + \alpha_{12} x X_2$$

$$\delta_1 = \delta_{10} + \delta_{11} x X_1 + \delta_{12} x X_2$$



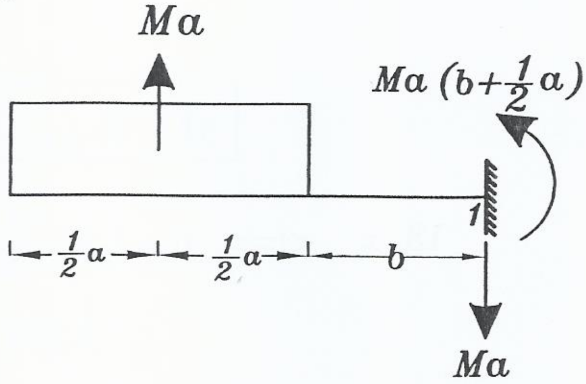
Main system(0)

B.M.D



Conj. Beam

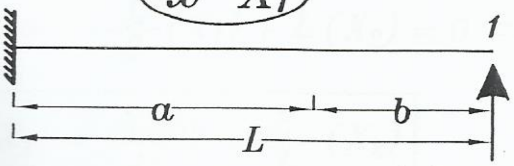
$\overline{EI}$



$$\alpha_{10} = \frac{1}{EI} x\, Ma$$

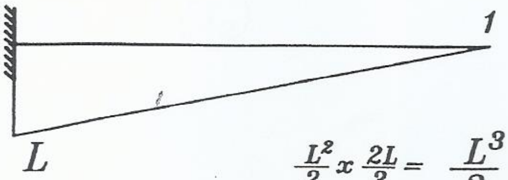
$$\delta_{10} = \frac{1}{EI} x\, Ma\,(b+\frac{1}{2}a)$$

$x\ X_1$



Correction system(1) 1t

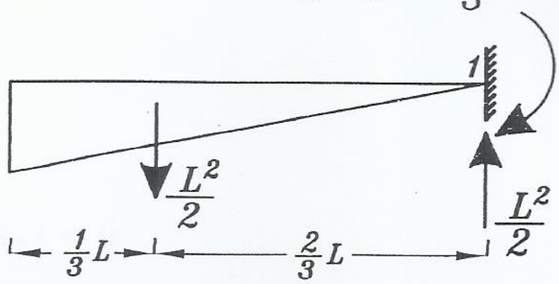
B.M.D



$\frac{L^2}{2} x \frac{2L}{3} = \frac{L^3}{3}$

Conj. Beam

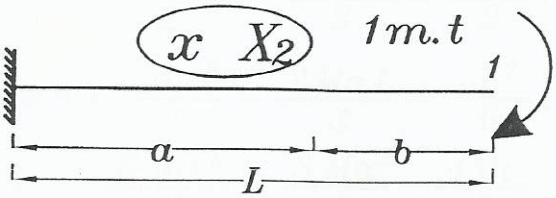
$\overline{EI}$



$$\alpha_{11} = \frac{-1}{EI} x \frac{L^2}{2}$$

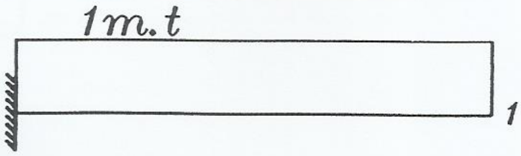
$$\delta_{11} = \frac{-1}{EI} x \frac{L^3}{3}$$

$x\ X_2$ 1m.t



Correction system(2)

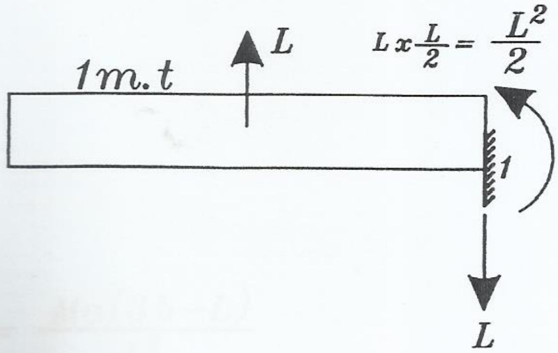
B.M.D



$L x \frac{L}{2} = \frac{L^2}{2}$

Conj. Beam

$\overline{EI}$



$$\alpha_{12} = \frac{1}{EI} x\ L$$

$$\delta_{12} = \frac{1}{EI} x \frac{L^2}{2}$$

$$\boxed{\alpha_1 = \alpha_{10} + \alpha_{11} x\ X_1 + \alpha_{12} x\ X_2}$$

$$\frac{1}{EI} x\, Ma + \frac{-1}{EI} x \frac{L^2}{2}(X_1) + \frac{1}{EI} x\ L\,(X_2) = 0 \implies x\ EI$$

$$Ma - \frac{L^2}{2}(X_1) + L\,(X_2) = 0 \implies EQ.1$$

$$\boxed{\delta_1 = \delta_{10} + \delta_{11} x\ X_1 + \delta_{12} x\ X_2}$$

$$\frac{1}{EI} x\ Ma\,(b + \tfrac{1}{2}a) + \frac{-1}{EI} x \frac{L^3}{3}(X_1) + \frac{1}{EI} x \frac{L^2}{2}(X_2) = 0 \implies x\ EI$$

$$Ma\,(b + \tfrac{1}{2}a) - \frac{L^3}{3}(X_1) + \frac{L^2}{2}(X_2) = 0 \implies EQ.2$$

From equation (1)

$$Ma - \frac{L^2}{2}(X_1) + L\,(X_2) = 0 \implies \frac{L^2}{2}(X_1) = Ma + L\,(X_2)$$

$$\boxed{X_1 = \frac{2Ma}{L^2} + \frac{2}{L}(X_2)}$$

Substitute in equation (2)

$$Ma\,(b + \tfrac{1}{2}a) - \frac{L^3}{3}(X_1) + \frac{L^2}{2}(X_2) = 0$$

$$Ma\,(b + \tfrac{1}{2}a) - \frac{L^3}{3}\left[\frac{2Ma}{L^2} + \frac{2}{L}(X_2)\right] + \frac{L^2}{2}(X_2) = 0$$

$$Ma\,b + \frac{Ma^2}{2} - \frac{2MaL}{3} - \frac{2L^2}{3}(X_2) + \frac{L^2}{2}(X_2) = 0$$

$$Ma\,b + \frac{Ma^2}{2} - \frac{2MaL}{3} - \frac{L^2}{6}(X_2) = 0$$

$$X_2 = \frac{6\,Ma\,b}{L^2} + \frac{3\,Ma^2}{L^2} - \frac{4Ma}{L}$$

$$= \frac{6\,Ma\,b}{L^2} + \frac{3\,Ma(L-b)}{L^2} - \frac{4Ma}{L}$$

$$= \frac{6\,Ma\,b}{L^2} + \frac{3\,Ma}{L} - \frac{3\,Ma\,b}{L^2} - \frac{4Ma}{L}$$

$$= \frac{3\,Ma\,b}{L^2} - \frac{Ma}{L} = \frac{3\,Ma\,b}{L^2} - \frac{MaL}{L^2} = \frac{Ma(3b-L)}{L^2}$$

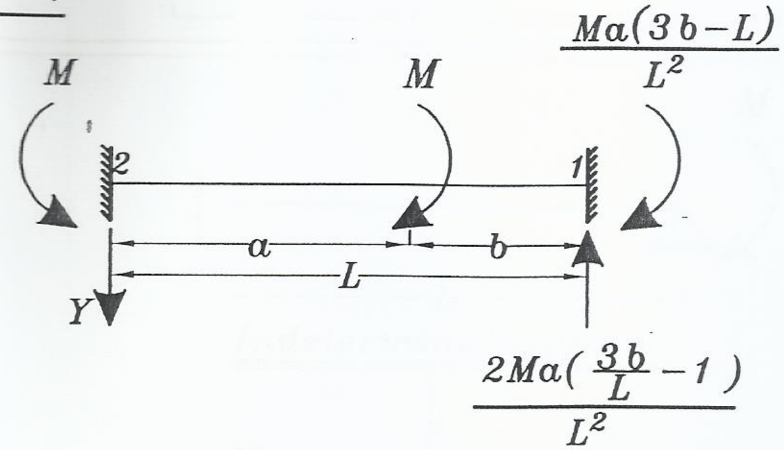
$$X_2 = \frac{Ma(3b-L)}{L^2}$$

Substitute in equation (1)

$$X_1 = \frac{2Ma}{L^2} + \frac{2}{L}(X_2)$$

$$= \frac{2Ma}{L^2} + \frac{2}{L}\left[\frac{Ma(3b-L)}{L^2}\right]$$

$$\therefore \quad = \frac{2Ma(\frac{3b}{L}-1)}{L^2}$$

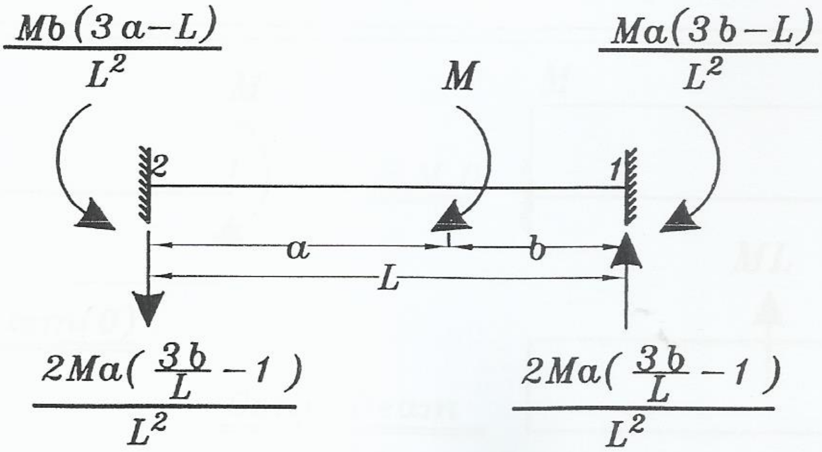


$$\Sigma M@2 = 0$$

$$M = \frac{Mb(3a-L)}{L^2}$$

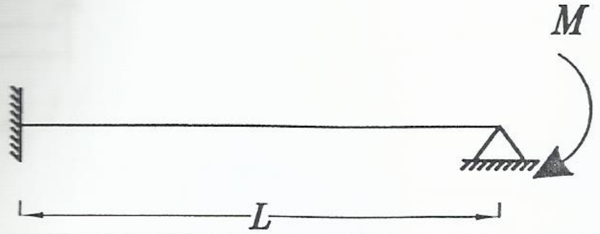
$$\Sigma Y = 0$$

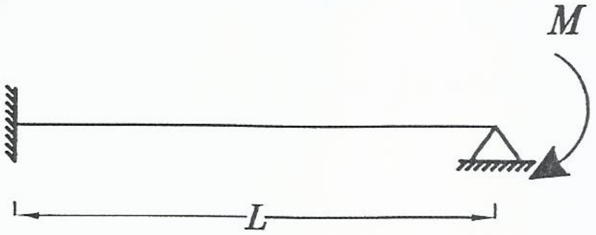
$$Y = \frac{Ma(\frac{3b}{L}-1)}{L^2}$$



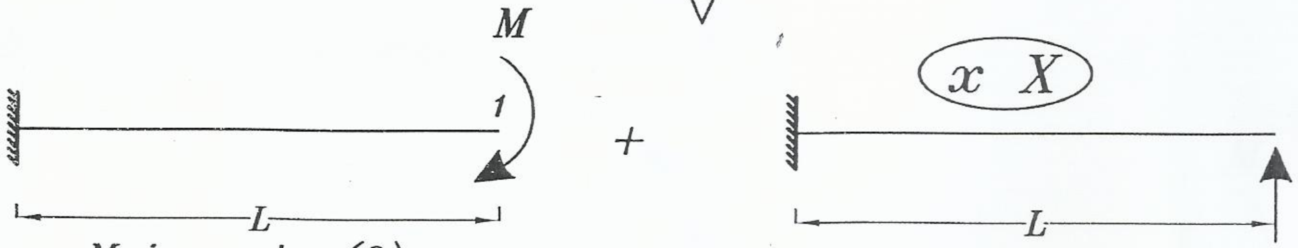
Example:

For the shown beam draw the B.M.D and calculate the reactions



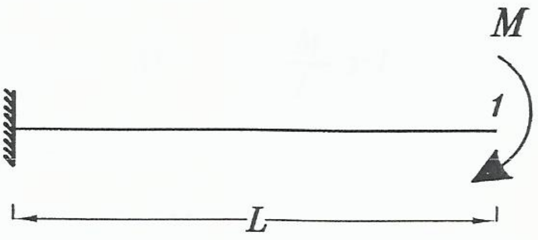


Indeterminate Beam



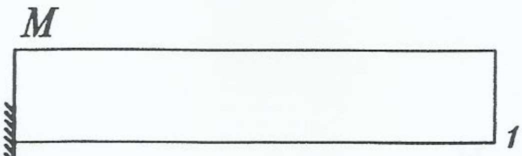
Main system(0) *Correction system(1)*

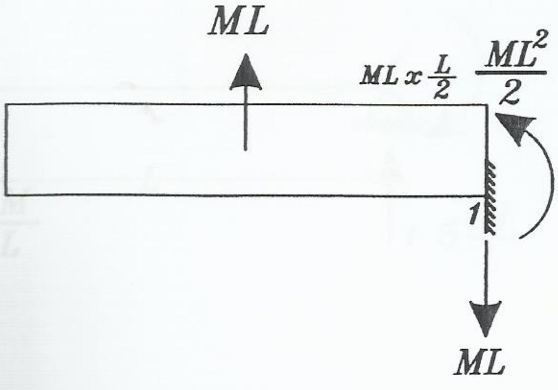
$$\delta_1 = \delta_{10} + \delta_{11} x\ X$$



Main system(0)

B.M.D



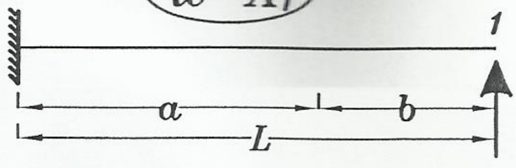


Conj. Beam

$\frac{M}{EI}$

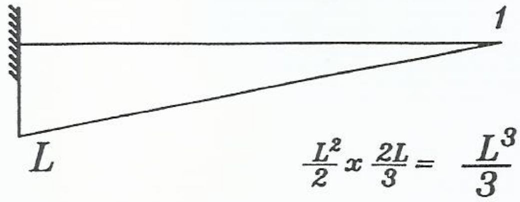
$$\delta_{10} = \frac{1}{EI} x \frac{ML^2}{2}$$

$x \; X_1$



Correction system(1)

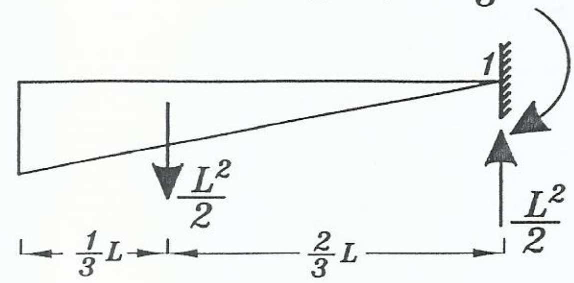
B.M.D



$\frac{L^2}{2} x \frac{2L}{3} = \frac{L^3}{3}$

Conj. Beam

$\overline{EI}$

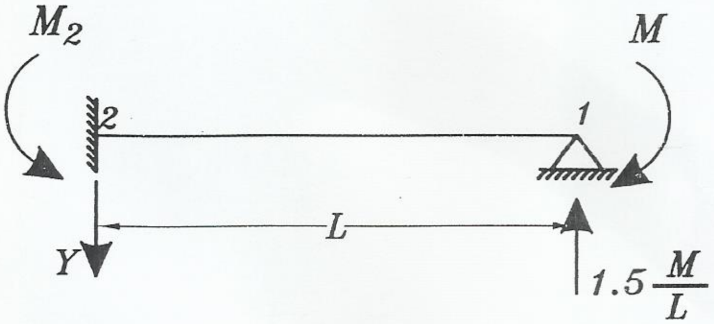


$$\delta_{11} = \frac{-1}{EI} x \frac{L^3}{3}$$

$$\boxed{\delta_1 = \delta_{10} + \delta_{11} x \; X}$$

$$\frac{1}{EI} x \frac{ML^2}{2} + \frac{-1}{EI} x \frac{L^3}{3}(X) = 0 \Longrightarrow x \; EI$$

$$\frac{ML^2}{2} - \frac{L^3}{3}(X) = 0 \Longrightarrow X = 1.5\frac{M}{L}$$



$\Sigma \; M@2 = 0$

$M_2 = M - 1.5\frac{M}{L} x \, L = -0.5M$

$\Sigma \; Y = 0$

$Y = 1.5\frac{M}{L}$